How to Assess Crack Failure

Crack propagation might spell doom, but FEA software and FAD analysis is able to tell users how much longer damaged equipment can safely stay in operation.

Motion-control systems provide greater control and accuracy for production. Laser-micromachining processes are useful when developing a prototype part. Once it comes time to mass-produce a product, the integration of human-machine interfaces paired with motion-control devices like servos help ensure accuracy and repeatability. Let’s review some basics of laser micromachining and then dive into how motion-control systems can improve the process.

Cracks found in machine and structural components raise two questions: Will the crack cause an immediate failure, and if not, how long until the crack grows large enough for failure?

To address the first question, two methods are used to evaluate cracked components:

- Ductile tearing analysis
- Failure Assessment Diagram (FAD)

Computing critical flaw sizes helps to ensure damage-tolerant components and set appropriate inspection schedules when combined with fatigue analysis. The historic Shippingport reactor vessel (Fig. 1) provides an example to compare the two analysis methods.

A crack in a pressure vessel is analyzed with quarter symmetry to simplify the model. The top head and flange are omitted, since the circumferential flange is much stiffer than the region around the nozzle. A stress concentration by the base of the nozzle is a good location to examine a postulated crack.

**CRACK MESHES**

3D crack meshes are generated for a range of crack sizes (FEACrack 2015). Figure 2 shows cutaway views of the surface cracks.
crack face for three crack sizes. The surface crack sizes are denoted by the crack length ($2c$) and the crack depth ($a$). A ductile tearing analysis requires a range of crack sizes to compute the tearing modulus, so the crack length-to-depth aspect ratio is kept constant at $2c/a = 3$ to create a range of crack sizes. Other crack aspect ratios can be used to develop a full trend of critical crack sizes for shorter to longer crack shapes.

The concentric and radial mesh lines from the crack front are needed to compute the J-integral. For elastic-plastic analysis, there is a set of initially coincident nodes for the collapsed brick elements at each crack front position. The coincident nodes can separate as the load increases, helping to capture crack front blunting.

For the elastic-plastic analysis, the A508 steel material yield strength is 36 ksi, and the tensile strength is 70 ksi (ASME 2010). A Ramberg-Osgood equation is fit to the two tensile values to obtain a stress-strain curve. The Young’s modulus ($E$) is 30,000 ksi, and the Poisson’s ratio is 0.29. The shell’s outside diameter is 125.75 in., and the thickness is 8.375 in.

FINITE ELEMENT ANALYSIS (FEA) RESULTS

The elastic-plastic analysis for each crack size uses 20 analysis steps for equilibrium convergence and to provide output sets at each pressure load. The principal stress results were presented for the largest and smallest crack sizes. The crack front J-integral values are shown in Figure 3. The plot x-axis is the crack front node position angle, starting at the bottom crack tip; the center of the plot is at the deepest point of the surface crack. As the crack size increases, so do the J values, which is typical for pressure loading causing tensile stress through the vessel thickness.

DUCTILE TEARING INSTABILITY ASSESSMENT

The ductile tearing instability analysis method (Anderson 2005, API 2007, Rowson 2011, and Thorwald 2011) requires a material resistance J-R curve and J-integral results for several crack sizes. The A508 steel J-R curve from API 579 Table F.10 is shown in Figure 4. The J-R curve is described by the power-law equation:

$$I_R = C_T (\Delta a)^{n_T} = C_T (a - a_0)^{n_T}$$

where $\Delta a$ is the crack extension measured from starting crack size $a_0$ to the current crack size $a$; $C_T$ is the tearing coefficient; and $n_T$ is the tearing exponent. For A508 steel at 550°F, the coefficient and exponent values are: $C_T = 3.443$ ksi*in, $n_T = 0.329$.

The 0.2 mm offset line shown in Fig. 4 intersects with the J-R curve at the $J_{IC}$ single toughness value, where crack front blunting gives way to ductile tearing behavior. The slope of the 0.2-mm offset line is given by a constant (default of 2 used here) multiplied by the flow stress. The intersection of the J-R curve and offset line is found by iteration.
The curves in Figure 5 present a comparison of the A508 J-R curve to other materials. Most are generic carbon and stainless steels from API 579, and the A53 curve is from material testing (Rowson 2011) according to the ASTM E1820 standard (ASTM 2015).

The ductile tearing instability assessment uses the crack front J-integral values at a given load and compares them to the material resistance J-R curve. Figure 6 shows the crack front maximum J values, which occur near the crack tip, for three pressure load cases versus a range of crack sizes, as represented by the crack depth a (plot x-axis), since the $2c/a$ aspect ratio is kept constant. The label J_{app} is the “applied” J value from the analysis results.

The J_{app} result curves in Fig. 6 are obtained by plotting J at the same load value from each crack analysis to get the trend of J versus crack size for comparison to the J-R resistance curve.

A starting point for a ductile tearing assessment would use at least three crack sizes—small, medium, and large—to obtain the J versus a trend and compute a tearing modulus. Adding more crack size cases is beneficial because it adds more data points to the curves, ultimately improving the trends and instability assessment.

Imagine the J-R curve in Fig. 6 being shifted left and right along the x-axis by varying $a_0$ until a tangent point is found with a J_{app} curve. For the maximum pressure load case, a tangent point is found at $J = 3.7$ ksi''in, corresponding to a starting critical crack size of $a_0 = 5.2$ by $2c_0 = 15.7$ in.

If the starting crack size is slightly larger, the crack would be unstable, predicting a failure at this pressure. In this case, the J_{app} and J-R curves would not intersect. If the starting crack size is slightly smaller, the crack would have stable tearing, and the J_{app} and J-R curves would intersect. Note that the two lower-pressure load cases intersect the J-R curve and would be considered stable for this starting crack size.

There can be a region of overlap in the J versus a curves near the tangent instability point, making visual identification of the tangent point somewhat imprecise. To more precisely locate the tangent instability point, the non-dimensional tearing modulus (T) is computed using:

$$T = \frac{E}{\sigma_y} \frac{dJ}{da}$$

where the tearing modulus is normalized by the Young’s modulus E and the yield strength; and $dJ/da$ is the derivative of the J_{app} or J-R curve. T_{app} is the tearing modulus computed from J_{app}, and T-R is the tearing modulus computed from the J-R curve.

For the J_{app} curve, a polynomial curve-fit works well to fit the J versus a results and obtain the $dJ/da$ derivative. The T_{app} tearing modulus is computed for the three pressure load cases at each crack size a and plotted versus the corresponding J values for those crack sizes in Figure 7.

In Fig. 7, points below the T-R curve are stable, and points above the T-R curve are unstable and predict failure. The
The intersection of the maximum pressure load \( T_{\text{app}} \) and T-R curves is where the slope of those curves is equal (tangent point), and the intersection gives a more definite instability point than on the J versus \( a \) plot. For the maximum pressure load case, the critical \( J = 3.70 \) ksi*in, which is found by iteration.

The critical J value from the tearing modulus curve intersection gives the y-axis value of the tangent instability point in Fig. 6. Solving the \( J_{\text{app}} \) curve-fit by iteration gives the crack size \( a \) at the tangent point on the \( J_{\text{app}} \) curve. Finally, Equation 1 is solved iteratively for the starting crack size \( a_0 \) using crack size \( a \) and the critical J value at the tangent point.

**Comparison to the Failure Assessment Diagram Method**

The FAD method (API 2007, Anderson 2005, Tipple 2012, and Thorwald 2016) evaluates a cracked structural component using two non-dimensional ratios—Kr and Lr—to give the assessment point location compared to the FAD curve.

The Kr ratio is the crack front stress intensity (\( K_I \)) computed using an elastic analysis, divided by the material fracture toughness (\( K_{IC} \)). The Lr ratio is the crack front reference stress computed using the J-integral from an elastic-plastic analysis divided by the yield strength. The reference stress calculation relies on the ratio of the total J divided by an extrapolated elastic J to compute both the Lr ratio and the analysis-specific FAD curve. If an Lr, Kr assessment point is below the FAD curve, it is stable; a point above the curve is unstable and predicts failure.

To compare the FAD method to the ductile tearing instability method, the assessment point is computed for the critical crack size obtained from the tearing instability (\( a = 5.2 \) by \( 2c = 15.7 \) in.). The \( K_{IC} \) toughness used to compute the Kr ratio is taken from the J-R curve in Fig. 4 at the 0.2-mm offset line.

Figure 8 shows that the assessment point (black circle) is outside the FAD curve, indicating that the FAD method is more conservative than the tearing instability method. This is expected since the J-R curve has a rising toughness trend to account for stable ductile tearing before failure, while the FAD method uses the single toughness value. The assessment point is near the right side of the FAD, and is to the right of the A508 recommended Lr\(_{\text{max}} = 1.25 \) cutoff, indicating that plastic collapse is a possibility. If the default Lr\(_{\text{max}} \) cutoff is used, the assessment point is still to the left of that plastic collapse limit.

Another way to compare the assessment methods is to compute the critical flaw size using the FAD method and compare that directly to the critical crack size obtained using tearing instability. By varying the crack sizes, the assessment points for critical crack sizes will be located on the FAD curves. One choice is to determine the critical flaw size on the analysis-specific FAD, and another choice is to determine the critical flaw size on the default API 579.
both methods. A range of crack sizes was used in both meshes computed the crack front J-integral values needed for reactor pressure vessel. Elastic-plastic FEA using 3D crack meshes to extend this analysis.

The FAD method needs high-enough crack front plasticity to complete the reference stress calculations, which can cause FEA convergence difficulties if the estimated maximum loading is too high. Often, the needed maximum load is above the assessment load (a design or operating load) and may take some trial-and-error for adequate J plasticity and convergence. The ductile tearing method only needs convergence up to the assessment load of interest, and does not have a crack front plasticity requirement.

For smaller cracks or lower loads, the ductile tearing method may be able to indicate quickly that the crack is stable from just a few analysis cases. A separate plastic collapse check is needed for the ductile tearing assessment, especially when the remaining ligament is small, whereas the FAD method has a built-in plastic collapse check at the Lrmax cutoff. The FAD and ductile tearing methods from API 579 are single-parameter assessment methods using K or J to characterize the crack front. In some crack cases, a two-parameter approach may be needed to characterize the crack front, in order to also compute, say, the T-stress. ASTM 2899 (see reference 13) uses a function of J and T-stress to determine the instability location along the crack front, which could be used to extend this analysis.

SUMMARY

The ductile tearing instability assessment and Failure Assessment Diagram methods were compared by computing a critical flaw size of a postulated crack in the Shippingport reactor pressure vessel. Elastic-plastic FEA using 3D crack meshes computed the crack front J-integral values needed for both methods. A range of crack sizes was used in both methods to determine the critical flaw size.

The tearing instability assessment uses the J-R material resistance curve, and the FAD method uses a single toughness value. In this example, the tearing instability gave a larger critical crack size compared to the FAD method. Both methods have advantages depending on the material behavior, the desired failure assessment method, and the material data available.

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