Crack front stress intensity validation using two methods for a crack at a material boundary in a nozzle component

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Abstract: To evaluate if a crack might cause a structural failure, the crack front stress intensity is needed according to engineering standards like API 579-1/ASME FFS-1. The objective of this paper is to describe how the computed crack front stress intensity can be validated by using two separate approaches with Abaqus/Standard. This paper examines the details of modeling a crack located at a material boundary in a nozzle component, including the effect of different modulus values on the computed stress intensity. Being able to compute the stress intensity two ways and get the same value gives confidence that the mesh is sufficient to model the crack and provide accurate values for the fracture assessment. The first approach uses the J-integral in Abaqus/Standard with a focused crack mesh, which is often the more convenient approach to obtain the stress intensity. The second approach uses the crack face opening displacement as a function of distance from the crack to directly compute the stress intensity at the crack front, which requires specific features in the mesh. Using Abaqus benefits engineers by allowing calculation of the crack front stress intensity for the specific crack location and the specific structural component geometry, avoiding the need to use an approximate stress intensity solution from a similar geometry.

Keywords: Surface crack, crack mesh, stress intensity, J-integral, material boundary, dissimilar metal, crack opening displacement, tied contact, nozzle.

1. Introduction

The crack front stress intensity, $K$, is needed for fracture assessment and fatigue crack growth calculations. One method to compute $K$ is to use the J-integral computed at the crack front nodes with the appropriate focused mesh. Another method to compute $K$ is to use the crack face opening displacements for several nodes at varying distances away from the crack front. Having two independent methods to calculate $K$ gives confidence that the crack mesh is sufficiently refined and appropriately constructed to model the crack and provide accurate $K$ values for the fracture assessment or fatigue analysis.

A pipe-to-nozzle geometry is used as an example to examine a crack at a material interface between dissimilar materials. One model of interest is a crack at the material boundary between a carbon steel pipe and stainless steel nozzle, where the stainless steel can have a lower modulus of elasticity, $E$, value. This example is used to examine the trends in the stress intensity values computed by the two methods. Another model of interest is for the crack within an average
material region, such as weld material, between the pipe and nozzle to investigate if this meshing approach can be used as an alternative to model a crack at the material boundary.

2. Nozzle models

The first nozzle model has a circumferential surface crack located at the material interface, so that one crack face is in the carbon steel pipe and the other crack face is in the stainless steel nozzle; see Figure 1. The mesh colors indicate the crack mesh region, material interface, and pipe and nozzle regions: blue and yellow mesh regions are carbon steel material, green and red mesh regions are stainless steel material. Figure 2 shows a close up of the crack at the symmetry plane and the focused mesh pattern at the crack front needed for the J-integral. In this half symmetric model, both crack faces are present, but just half of the total crack length, 2c, is present in the mesh. The nozzle (thicker right end) is the stainless steel material, and will be the region where the modulus value is varied in the analyses. The pipe (left end) is the carbon steel material, with the modulus held constant in the analyses. The Poisson ratio is held constant in both regions. The nozzle mesh is half symmetric about the x-y plane. The nozzle has internal pressure and an equivalent axial force applied to the pipe at the left end; the right end of the nozzle is constrained in the axial x-direction.

The custom crack mesh region is generated using the FEACrack™ software and connected to the surrounding mesh using tied contact (FEACrack, 2013). Connecting the crack mesh to the surrounding mesh by tied contact has worked very well in previous models (Tipple, 2012), especially when the element sizes are similar on each side of the tied surface. Figure 1 shows the tied contact locations between the green and red mesh zones in the nozzle, and between the yellow and blue mesh zones in the pipe. In Figure 3 the pipe region is removed to show the crack face mesh pattern. Note that the radial mesh lines near the crack front within the focused mesh region are perpendicular to the semi-elliptical crack front to facilitate the K from crack face opening displacement post-processing calculation.
Figure 1. Model 1, circumferential crack at material interface, half symmetric.

Figure 2. Model 1, close up of the crack and focused mesh region.
Figure 3. Model 1, cut away to show the crack face in the nozzle.

Figure 4. Model 2, circumferential crack in the average material zone.
The second nozzle model uses a narrow circumferential mesh zone between the pipe and nozzle regions so that the crack is within an average modulus value, see Figure 4 and Figure 5. The blue and yellow mesh regions are carbon steel material; the center green mesh zone is the average material, and the light blue and red mesh zones are stainless steel material. The average material zone modulus is the average of the constant carbon steel modulus and the varying nozzle modulus. The average material mesh zone may be appropriate in some cases where the joining of the dissimilar metals uses an intermediate material, such as a dissimilar metal weld (DMW), or as a possibly more convenient choice to creating the crack mesh.

Generic dimensions, properties and pressure load are used for these nozzle models to compare K result trends; the model values are given in Table 1.

### Table 1. Model data.

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<table>
<thead>
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<tbody>
<tr>
<td>Inside radius</td>
<td>3.0 in</td>
</tr>
<tr>
<td>Pipe thickness</td>
<td>1.0 in</td>
</tr>
<tr>
<td>Nozzle thickness</td>
<td>2.0 in</td>
</tr>
</tbody>
</table>
Center mesh zone 0.5 in
Crack length, 2c 4.0 in
Crack depth, a 0.5 in
Internal pressure, P 0.25 ksi
Pipe modulus, $E_1$ 30000 ksi
Nozzle modulus, $E_2$ varies from 30000 ksi to 25000 ksi
Poisson ratio, $\nu$ 0.3

3. K equations

The equations needed to relate the J-integral and crack face opening displacement to the crack front stress intensity, $K$, are given below.

The Abaqus User’s Guide, section 11.4.2 provides the equations to relate the J-integral to $K$ for a crack at a material interface (SIMULIA, 2013).

$$J = \frac{1-\beta^2}{E^*} (K_I^2 + K_{II}^2) + \frac{1}{2G^*} K_{III}^2$$ \hspace{1cm} (1)

where $J$ is the J-integral, $K_I$, $K_{II}$, $K_{III}$ are the three stress intensity factor modes for general crack opening, $E^*$ is the combined modulus given in Equation 3, $G^*$ is the combined shear modulus given in Equation 5, and $\beta$ is given by Equation 2.

$$\beta = \frac{G_1(\kappa_2-1) - G_2(\kappa_1-1)}{G_1(\kappa_2+1) + G_2(\kappa_1+1)}$$ \hspace{1cm} (2)

where $G_1$ and $G_2$ are the shear modulus for the two materials on each side of the interface: the carbon steel pipe and stainless steel nozzle for this example; $\kappa_1$ and $\kappa_2$ are given for each material by Equation 7 for the case of plane strain.

$$\frac{1}{E^*} = \frac{1}{2} \left( \frac{1}{E_1} + \frac{1}{E_2} \right)$$ \hspace{1cm} (3)

where $E^*$ is the combined modulus, $E_1$ and $E_2$ are given by Equation 4 for each material at the interface, and $E$ is given by Equation 4 in terms of the modulus of elasticity, $E$, and the Poisson ratio, $\nu$, for the case of plane strain.
\( E = \frac{E}{(1+\nu^2)} \)  \hspace{1cm} (4)

\( \frac{1}{G^*} = \frac{1}{2}\left(\frac{1}{G_1} + \frac{1}{G_2}\right) \)  \hspace{1cm} (5)

where \( G^* \) is the combined shear modulus at the material interface; \( G_1 \) and \( G_2 \) are the shear modulus for the two material values at the interface; \( G \) is given in terms of the modulus of elasticity and Poisson ratio in Equation 6.

\( G = \frac{E}{2(1+\nu)} \)  \hspace{1cm} (6)

For plane strain, \( \kappa \) used in Equation 2 is given in terms of the Poisson ratio.

\( \kappa = 3 - 4\nu \)  \hspace{1cm} (7)

Equation 1 can be simplified when the crack is in a single material zone, which avoids the \( E^* \), \( G^* \) and \( \beta \) terms, and if just crack opening mode I is expected, and for the case of plane strain. The simplified J equation in terms of the mode I stress intensity is given in Equation 8.

\[ J = \frac{(1+\nu^2)K_I^2}{E} \]  \hspace{1cm} (8)

The second method used to compute the stress intensity uses the crack face opening displacements, and is given by Equation 9 (Anderson, 2005).

\[ K_I = 0.25E \frac{U_{total}}{2} \left(\frac{2\pi}{R}\right)^2 \]  \hspace{1cm} (9)

where \( K_I \) is the mode I crack opening stress intensity due to the \( U_{total} \) crack face opening displacement between the crack faces at radial distance \( R \) from the crack front. Crack opening displacement equations are available for mode II and mode III, but are not needed for this example, since the results indicate nearly pure mode I crack opening. Since the crack front node is at distance \( R = 0 \), it would cause a division by zero and cannot be used in Equation 9. Instead, crack face nodes away from the crack front along a radial mesh line are used to obtain a trend of stress intensity versus distance.

Using the equations for these two methods, the stress intensity from J and the stress intensity from the crack face opening displacement can be compared.
4. K from crack face opening displacement

After running the Abaqus/Standard analysis for both nozzle models, the crack face opening displacements are obtained for a set of nodes at varying distance away from the crack front. Figure 6 shows the axial stress and deformed shape of Model 1 with the crack at the material interface; the displacement scale is 5000x to see the crack opening. Figure 7 shows a close up of the crack front and the location of the crack face radial nodes used to compute K from the crack opening displacement.

![Figure 6. Axial stress results, crack face opening; 5000x displacement scale.](image)

Using Equation 9, the stress intensity values are computed versus each node’s distance, R, from a crack front node. Figure 8 shows the plot of K versus distance. For this example, the trend appears linear, so a straight-line curve-fit was used to extrapolate the K value trend to the crack front node at distance R = 0, which is essentially the y-intercept value of the linear-fit. The K from displacement calculation is repeated at the other crack front nodes so that the K from displacement can be compared to the K from J values along the crack front. The first K comparison has the modulus of elasticity equal on both sides of the crack front (case 1 in the plots); in Figure 9 the K values agree closely along the crack front except at the crack tip. The crack tip K from J value is often not as close to the K from displacement as the rest of the crack front since the crack tip element can have a more triaxial stress state and also uses the plane stress
version of the K from J equation. The overall crack front agreement indicates that both K calculation methods agree.

Figure 7. Close up of the crack face opening and radial nodes; 5000x scale.
Figure 8. K from crack face displacement trend extrapolated to the crack front.

K from displacement: extrapolate crack face trend to R=0

\[ y = 0.048185x + 0.512511 \]

\[ R^2 = 0.976858 \]
Figure 9. Compare K from displacement to K from J along the crack front; equal modulus values on each side of the crack.

5. Compare K results

The crack depth location is used to compare more K results for the other nozzle modulus values. The original model of interest was a stainless steel nozzle material, with a lower modulus value of 28000 ksi (case 3 in the plots), and was varied to 25000 ksi to examine the K results trend. The y-axis of the plot in Figure 10 is normalized by the K value in case 1, when the modulus values are equal, as a way to show the percent difference in the K results. The K trend shows relatively close agreement for the stainless steel modulus, and increasing difference, still less than 1%, in the K results as the nozzle modulus value is reduced to 25000 ksi.

The plot in Figure 11 compares the K results for Model 2 where the crack is in the average material mesh zone. The average modulus is computed as the simple average of the constant pipe modulus and the varying nozzle modulus. The agreement between K results is very close,
possibly since a single modulus value is used to compute $K$ from $J$ using Equation 8. Both Figure 10 and Figure 11 show that $K$ from $J$ agrees with $K$ from displacement when the modulus values are equal.

The $K$ trend also shows there is just a few percent increase in the $K$ value as the nozzle modulus is reduced, since the lower modulus allows more crack face opening displacement, which increases the $K$ value.

![Graph](image)

**Figure 10. Model 1 crack at material interface.**
Figure 11. Model 2 crack in average material zone.
Figure 12. Compare models; K from crack opening displacement, U.

The plot in Figure 12 compares the K from crack face opening displacement, U, for the two models; the y-axis is normalized by the Model 1 K from displacement value for case 1 with equal modulus values. The trend shows close comparison in the K results, especially for the original model of interest with the nozzle having the stainless steel modulus value. This trend indicates that both models provide very similar results, so the average material mesh zone could be used if it was more convenient than the crack at the material interface when creating the mesh, or when there is an intermediate material between the pipe and nozzle, such as a dissimilar metal weld.

6. Summary

Two nozzle models with a circumferential crack at a material interface were used to compare the stress intensity, K, results computed by two methods: K from the J-integral, and K from the crack face opening displacement. The K results agree for the equal modulus case, and agree closely as...
the nozzle modulus is reduced to model dissimilar metal in the pipe and nozzle. The $K$ results also agree closely for the models with the crack at the material interface and with the crack in an average material mesh zone. Having two independent methods to compute $K$ allows the $K$ results and the crack mesh to be validated, and gives confidence in computing accurate $K$ values for fracture assessments and fatigue analyses. The close agreement in the $K$ results for the two models shows that there is flexibility in having the crack at the material interface or within an average material mesh zone between the materials.

7. References

2. FEACrack, Version 3.2.24, 2013, Quest Integrity Group, Boulder, Colorado.